Intermittency interpretation of persistent small scale anisotropy in turbulence

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A model of small scale anisotropy in turbulence is motivated by the proposal that intermittency properties in turbulence are governed by direct coupling between the largest scales and inertial range length scales. A one-parameter hierarchy of scalar derivative moment dependences on Reynolds number Re is predicted. Enforcing the observed constancy of the skewness, divergences of higher moments are predicted. The results support the proposed extremal nature of intermittency scalings. [S1063-651X(96)51006-8]

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The persistent skewness of scalar derivatives in turbulence, in violation of the isotropy expected at high Reynolds number Re, is a long-standing conundrum [1]. (Most reported observations involve passive scalars, but analogous results were recently obtained for a vorticity component [2].) This behavior suggests that the direct influence of large scales is nonnegligible throughout the inertial range.

Direct coupling (DC) between the largest scales and inertial range length scales has recently been proposed as the mechanism underlying the intermittency corrections to the velocity structure-function exponents [3]. This picture is the basis of several models whose predictions are distinct, yet none of them are ruled out by existing experimental results [4–6].

In addition to the support for DC implied by the success of the structure-function analysis, there is more explicit evidence of DC effects. Analysis of triadic interactions in direct numerical simulations and stochastic models of turbulence indicates that a ''distant interaction'' mechanism (a particular form of DC) may play a significant role over arbitrarily large scale separations [7].

In this context, consider the mechanisms determining the fluctuations of a passive scalar in turbulence subject to an imposed mean scalar gradient G. Homogeneous isotropic turbulence is considered here for simplicity, although the behaviors of interest are seen also in shear flows. Conventional reasoning based on local (in wave number) transfer of scalar spectral intensity suggests that the imposed scalar derivative bias is overwhelmed by the total fluctuation intensity at high Re, causing the derivative skewness to vanish in the high Re limit.

An alternative picture based on DC is as follows. Intense events cause regions whose size is order L, where L is the integral scale, to be compressed to length scale $r \ll L$, where r is any inertial range length scale. (The events must also involve extensive strain to satisfy incompressibility.) It must also be assumed that at least some of these events are nearly irrotational, so that the alignment of the local scalar gradient vector with the mean gradient is preserved to some extent.

The character of fluid motions corresponding to these events has been investigated [1,8,9]. The best evidence is provided by simulations of three-dimensional Navier-Stokes turbulence [9]. It is found that compression-dominated regions adjacent to tubes of intense vorticity are sufficiently intense and irrotational to cause the observed scalar gradient amplification.

The nature and origin of these fluid motions is immaterial to the present analysis. Their existence is viewed here as a necessary implication of the anisotropy observed high Re. The consequences of their existence are considered.

The DC analysis of structure functions demonstrates that application of scaling hypotheses to the most intense events in turbulence can yield useful predictions [3]. Here, a simple scaling hypothesis is applied to the nearly irrotational events responsible for persistent anisotropy. Though there is some commonality of viewpoint with the structure-function analysis, there is no evident link between the mathematical framework of that analysis and the present problem. Moreover, the two problems are governed by distinct types of intense events that may obey different scalings.

The scaling hypothesis adopted here is motivated by the following physical picture. Consider the DC contribution to scalar fluctuations at length scale r. The net contribution is viewed as a balance between an input due to direct transfer of fluctuation intensity from length scale L and dissipation of those fluctuations by the conventional eddy cascade mechanism, governed by the eddy time scale τ_r. (τ_r scales as $r^{2/3}$ in the inertial range [10], but this is immaterial here.)

The net DC contribution to fluctuations at scale r is characterized by a probability distribution $f(r)$ such that $f(r)dr$ represents the mean volume fraction in which the scalar fluctuations are dominated by the effect of nearly irrotational direct transfer from L to the length scale range $(r,r+dr)$. Denoting the cumulative distribution as $F(r)=\int_0^r f(r')dr'$, $F(L)$ represents the total volume fraction in which scalar fluctuations are dominated by DC contributions.

Scaling of $f(r)$ is assumed, namely $f(r)\sim r^p$, where p is a free parameter. It is also assumed that the scalar derivative in the mean-gradient direction in regions dominated by direct transfer from L to r is of order GL/r. This follows from the relation $G=\Delta/L$, where Δ is the mean scalar increment over a distance L in the gradient direction. Compression of a size-L region to size r causes a size-Δ scalar increment, originally spanning a distance L, to span instead the smaller distance r. The corresponding gradient is $\Delta/r=GL/r$. Here, r has any non-negative value less than L; the fine scale viscous cutoff of the inertial range has not yet been invoked.

Now, the Kolmogorov microscale $\eta\sim \mathrm{Re}^{-3/4}L$ governing the viscous cutoff of the inertial range is introduced [10].

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Consider an intense event inducing direct transfer from L to $r \ll \eta$. Physical principles do not uniquely prescribe its fate, but two plausible scenarios yield identical scaling predictions. One scenario is that further compression is arrested when the length scale in the compressive strain direction is reduced to η. In effect, the L-to-r compression event is truncated to an L-to-η event. Another scenario is that conventional Kolmogorov scaling does not apply during the event, but takes effect upon completion of the event. In this case, compression to length scale r does occur, but its effect on scalar derivative fluctuations dissipates on a time scale r^2/ν, where ν is the kinematic viscosity, rather than τ_r. For $r \ll \eta$, r^2/ν is much less than τ_r. (Indeed, η is defined as the length scale at which these time scales balance. In this discussion, unity Prandtl number Pr is assumed for simplicity.) This implies accelerated dissipation. The least singular behavior possible within this scenario is assumed, namely the annihilation of any contribution of $r < \eta$ events to the derivative statistics.

These two scenarios, conveniently denoted as the ''truncation'' and ''annihilation'' scenarios, respectively, are subsumed within a modified cumulative distribution $F(r) = F_0(r) + F_1(r)$. Under both scenarios, F_0 and F_1 are zero for $r < \eta$, and $F_0(r) = \int_\eta^r f(r')dr'$ for $r \geq \eta$, where f is defined as earlier (omitting η effects). F_0 is the cumulative distribution under the annihilation scenario, so for this scenario, $F_1(r) = 0$ for all r. Under the truncation scenario, $F_1(r) = \int_0^\eta f(r')dr'$ for $r \geq \eta$. It will be seen that the contributions of F_0 and F_1 to derivative moments obey the same scalings and thus that the scaling of F is not scenario dependent.

Based on the assumed scaling $f(r) \sim r^p$, the assumed dependences (under two scenarios) of F on f, and the assumed derivative magnitude GL/r for given r, the DC contribution to the jth derivative moment M_j is obtained from $M_j \sim [1/F(L)] \int_0^{F(L)} (GL/r)^j dF$. Mean-subtracted moments m_j can be expressed in terms of these moments. Both sets of moments obey the same scalings because moments diverge as powers of Re that increase in proportion to j (according to the model). Namely, F_0 and F_1 each yield the scaling $m_j \sim \eta^{p+1-j} \sim \mathrm{Re}^{3(j-p-1)/4}$, where the Re scaling of η has been invoked and $j > p+1$ has been assumed (subject to later verification). This result indicates the increasingly strong Re dependence with increasing j and also demonstrates the equivalence of the two scenarios for present purposes.

Normalized moments $m_j/m_2^{j/2}$ are considered for $j > 2$ (skewness, kurtosis, etc.). Note first that the conventional scaling of the variance m_2 appearing in the denominator is $m_2 \sim \mathrm{Re}$. To see this, note that the scalar dissipation obeys $\epsilon_\theta \sim \nu m_2$ [10]. (Recall that Pr=1 is assumed.) The Re sensitivity in the scaling analysis is based on variation of ν with fixed large scale flow properties. ϵ_θ is a large scale quantity and hence does not vary with Re. Therefore the relation $m_2 \sim \epsilon_\theta/\nu$ implies $m_2 \sim \mathrm{Re}$.

It will be seen that this conventional scaling dominates the DC contribution to m_2, consistent with the well established validity of the conventional result. Adopting the conventional scaling of m_2, the hierarchy of normalized moments is predicted to obey the scaling

$$\frac{m_j}{m_2^{j/2}} \sim \mathrm{Re}^{(j-3p-3)/4}, \tag{1}$$

involving one free parameter p.

To match the observed constancy of the derivative skewness at high Re [1,2,11], p is set equal to zero. Equation (1) then becomes

$$\frac{m_j}{m_2^{j/2}} \sim \mathrm{Re}^{(j-3)/4}. \tag{2}$$

Recall that the scaling of m_j is based on the DC contribution. For $j=2$, Eq. (2) is therefore the ratio of the DC to the conventional contribution. This ratio is seen to vanish as $\mathrm{Re}^{-1/4}$, consistent with the earlier claim that the conventional contribution is dominant.

For $j \geq 3$, Eq. (2) gives nonvanishing normalized moments in the high-Re limit. Conventional scaling is consistent with isotropy, and thus vanishing odd normalized moments, in this limit. Therefore, with p chosen to reproduce the behavior of the skewness, the predicted DC contributions to the odd normalized moments dominate the conventional contributions. Comparison of the measured Re dependences of higher order odd moments to Eq. (2) provides an unambiguous test of the DC model.

Conventional intermittency analysis does not uniquely prescribe the Re dependence of even derivative moments. For velocity derivative moments, two alternative scenarios prescribe distinct scalings that are not discriminated by the available experimental data [12]. Scalar derivative moment scalings are subject to this ambiguity, and in addition, the familiar complication of scalar-velocity correlation effects [13]. Thus, it cannot be determined whether the DC contribution predicted by Eq. (2) dominates the conventional contribution.

The best available data to test the predictions are the temperature derivative kurtosis measurements of Tong and Warhaft [11]. They infer $\mathrm{Re}^{0.2}$ dependence from their data, but the $\mathrm{Re}^{1/4}$ dependence predicted by Eq. (2) is plausible within the range of experimental uncertainty. They also report the derivative kurtosis normal to the mean temperature gradient. Presumably, this quantity reflects DC effects to a lesser degree, if at all, because fluid rotation is required to obtain a nonzero scalar derivative in this direction. This quantity is found to exhibit $\mathrm{Re}^{0.17}$ dependence, but Tong and Warhaft caution that the difference between this and $\mathrm{Re}^{0.2}$ dependence may not be statistically significant. The data indicate, however, that the prefactor of the scaling is significantly higher for the derivative in the mean gradient direction. On this basis, tentative support for DC dominance of the even moments may be inferred. More precise measurements of high derivative moments over a wide Re range are needed to obtain a definitive test of Eq. (2).

The empirically deduced result $p=0$ corresponds to an extremal behavior in the following sense. Consider a linear path of length L in the gradient direction. The mean difference GL between values of the scalar at the end points is the sum of a DC contribution S_D and a remainder S_0. The DC contribution is of order $\hat{S} \equiv \int_0^{F(L)} (GL/r) L dF$. This is obtained by integrating the slope GL/r times the measure LdF of the region of the path corresponding to the r interval $(r,r+dr)$.

The DC contribution S_D must be less than GL unless the contribution S_0 from outside the DC-dominated region is opposite in sign to G. The latter eventuality is implausible, the more so if the DC contribution diverges with increasing Re. Cancellation of this divergence by a concomitant divergence of S_0 would be required to obtain $S_D+S_0=GL$ for finite GL.

These considerations preclude the high-Re divergence of $\hat{S}$, therefore requiring the exponent p to be positive. For $p=0$, logarithmic divergence is obtained. $p=0$ is nevertheless admissible if the possibility of corrections to the proposed scalings is recognized. In any event, an implication of the model is that the contribution of DC effects to derivative moments is as large as physical realizability allows.

There is a noteworthy parallel between this analysis and a criterion proposed by Novikov to establish whether intermittency as manifested by structure-function scalings is as strong as mathematical constraints allow [4]. Experimental results to date do not provide a definitive answer [5,6]. The present analysis does not address this question specifically, but it does lend support to the notion that intermittency effects are extremal in some sense. The support for this notion is based on the observed constancy of the derivative skewness. The analysis does not identify the physical origin of this property.

The present analysis indicates that fine scale anisotropy may be the sum of two contributions of comparable magnitude, one specifically associated with the viscous cutoff scale, and another distributed across the inertial range, though strongest near the cutoff. This possibility complicates the interpretation of fine scale ramp-and-cliff features seen in measured [1,11] and numerically simulated [8,9] scalar profiles. These features scale with η, as anticipated, but this observation does not discriminate between the two proposed scenarios, nor does it rule out other scenarios.

By the same token, the present analysis is not helpful for interpreting the probability density function of scalar derivatives and related fluctuation statistics [8,9,11]. These quantities require a fluctuation model of DC, but a mean-field model has been proposed here. Owing to its phenomenological nature, it would be premature to extend this model to treat fluctuations prior to a definitive test of Eq. (2). Irrespective of the performance of the model proposed here, the key inference is that parameter dependences of small scale anisotropy can be predicted and interpreted in terms of the inertial range scaling properties of DC effects.

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